1А ОД 10.12

Да се докаже

$$\cos x + \cos^2 x + \ldots + \cos^n x = \underbrace{\frac{\sin\left(\frac{nx}{2}\right)\cos\left(\frac{(n+1)x}{2}\right)}{\sin\frac{x}{2}}}_{=A} \quad \text{(сокращение)}$$

и

$$\sin x + \sin 2x + \ldots + \sin nx = \underbrace{\frac{\sin\left(\frac{nx}{2}\right)\sin\left(\frac{(n+1)x}{2}\right)}{\sin\left(\frac{x}{2}\right)}}_{=B} \quad \text{(сокращение)}$$

$$z = A + iB, \quad z \in \mathbb{C}$$

$$z = \cos x + i\sin x + \ldots + \cos^n x + i\sin nx$$

полагаме $\cos x + i\sin x = t \Rightarrow$

$$z = t + t^2 + \ldots + t^n = \frac{t(t^n - 1)}{t - 1}$$

(сокращение)

$$(t + t^2 + \ldots + t^n)(t - 1) = t^{n+1} - t$$

$$z = t\,\frac{\cos nx + i\sin nx - 1}{\cos x + i\sin x - 1} \Rightarrow \cos 2x = \cos^2 x - \sin^2 x = \underline{1 - 2\sin^2 x}$$

полагаме

$$\Rightarrow t\,\frac{-2\sin^2\frac{nx}{2} + 2i\sin\frac{nx}{2}\cos\frac{nx}{2}}{-2\sin^2\frac{nx}{2} + 2i\sin\frac{x}{2}\cos\frac{x}{2}} \Big| \cdot\frac{i}{i} \Rightarrow$$

$$t\,\frac{-2i\sin\frac{nx}{2} - 2\sin\frac{nx}{2}\cos\frac{nx}{2}}{-2i\sin^2\frac{nx}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}} =$$

(1)

$$= (\cos x + i\sin x)\,\frac{\sin\frac{nx}{2}\left(\cos\frac{nx}{2} + i\sin\frac{nx}{2}\right)}{\sin\frac{x}{2}\left(\cos\frac{x}{2} + i\sin\frac{x}{2}\right)} =$$

$$= \frac{\sin\frac{nx}{2}}{\sin\frac{x}{2}}\left(\cos\left(\frac{nx - x + 2x}{2}\right) + i\sin\left(\frac{nx}{2} - \frac{x}{2} + x\right)\right) =$$

$$= \underbrace{\frac{\sin\frac{nx}{2}\cos\frac{(n+1)x}{2}}{\sin x/2}}_{\text{"A}} + i\underbrace{\frac{\sin\frac{nx}{2}\sin\frac{(n+1)x}{2}}{\sin\frac{x}{2}}}_{\text{"B}}$$

Нека $m, n \in \mathbb{N}$, n не дели m

$$\omega_k = \cos\left(\frac{2k\pi}{n}\right) + i\sin\left(\frac{2k\pi}{n}\right)$$

Докажете че:

** $\omega_0^m + \omega_1^m + \ldots + \omega_{n-1}^m = 0$

$\omega_0 = 1$

$\omega_1 = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$

$\omega_2 = \cos\frac{4\pi}{n} + i\sin\frac{4\pi}{n}$

ω_e за $e \geq 1$, $e \leq n-1$

$\omega_e = \omega_1^e$

$$\omega_0 + \omega_1^m + \omega_1^{2m} + \dots + \omega_1^{(n-1)m} = 1 + \omega_1^m + \omega_1^{2m} + \dots$$

Нека положим $\omega_1^m = t$ и от дясно

$$(t, t^2, t^3 \dots t^{n-1})(t-1) = t^n - 1$$

$$t = \omega_1^m = \cos\frac{2m\pi}{n} + i\sin\frac{2m\pi}{n} \neq 1$$

Заместваме в $(**)$

$$\frac{\omega^{mn}-1}{\omega^n-1} = \frac{\cos\frac{2mn\pi}{n} + i\sin\frac{2mn\pi}{n}}{\omega^n-1} = 0$$

Да се реши уравнението

$$x = \sqrt[8]{(1+i)^8}$$

$z_1 = 1+i$ в алгебричен вид

$$|z| = \sqrt{1+1} = \sqrt{2}$$

$$z = \sqrt{2}\left(\frac{1}{\sqrt{2}} + i\,\frac{1}{\sqrt{2}}\right) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

Тригонометричен вид на z

$$z^8 = 16\left(\cos\frac{8\pi}{4} + i\sin\frac{8\pi}{4}\right) = 16\left(\cos 2\pi + i\sin 2\pi\right)$$

$$z_k = \sqrt[8]{16}\left(\cos\frac{2k\pi+2\pi}{8} + i\sin\frac{2k\pi+2\pi}{8}\right)$$

$$z_0 = \sqrt[8]{16}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$z_1 = \sqrt[8]{16}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

Да се реши уравнението

$x^4 = \sqrt{3} + i$ / полагаме $x^4 = z$

$z = \sqrt{3} + i \Rightarrow |z| = \sqrt{3}+1 = \sqrt{3}+\sqrt{1} = \sqrt{4} = 2$

$$z = 2\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$\varepsilon_0 = \sqrt[4]{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$

$$\varepsilon_1 = \sqrt[4]{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$$

$$\varepsilon_2 = \sqrt[4]{2}\left(\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}\right)$$

$$\varepsilon_3 = \sqrt[4]{2}\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right)$$

$x^2 + 2x + 3 = 0$

$D = -8$; $x_{1,2} = \frac{-2 \pm \sqrt{D}}{2} = \frac{-2 \pm \sqrt{-8}}{2} = -1 \pm \sqrt{2}\,i$

полагаме $z = -2 = -2 + 0i \Rightarrow |z| = 2$

т.е. $z = 2(-1 + 0i) = 2(\cos\pi + i\sin\pi)$

от $x_{1,2}$ ±

$$\sqrt[2]{z} = \sqrt{2}\left(\cos\frac{2\pi+\pi}{2} + i\sin\frac{2\pi+\pi}{2}\right) = \pm i\sqrt{2}$$

$$x_1 = -1 + i\sqrt{2} \;/.(-1) = 1 - i\sqrt{2}$$
$$x_2 = -1 - i\sqrt{2} \;/.(-1) = 1 + i\sqrt{2}$$

Матрици

Правило
т-ред вънаги да е равно на п-стълб

I $\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{21} & \dots & \dots \\ 0 & 0 & a_{31} & \dots \\ 0 & 0 & 0 & \dots a_{mn} \end{pmatrix}$ - системата е определена

II $\begin{pmatrix} a'_{11}, a'_{12}, \dots a'_{1n} \\ 0\ 0\ 0\ 0 \dots \\ 0\ a_{32}\ 0\ 0\ 0 \dots \\ 0\ 0\ 0\ a_{nk} \dots a_{mn} \end{pmatrix}$ - системата е неопределена

III $\left(\begin{array}{l} a''_{11}, a''_{12}, \dots a''_{1n} \\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ \hline \dots \\ a_{1n} \dots a_{mn} \end{array}\right) \begin{array}{|l} b_i \\ b_i \end{array}$ - системата е несъвместима